MATH 3235 Probability Theory

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$X_n \to X$

We have a family of i.i.d. r.v. X_i

$$\overline{T}_N = \frac{1}{N}\sum_{i=1}^{N} X_i$$

If $X \geq 0$ with prob 1 and $\mathbb{E}(X) = 0 \Rightarrow X = 0$ prob 1

Suppose X_n is a sequence of r.v. and X is a r.v. if

$$\lim_{n\to\infty} \mathbb{E}\left((X_n - X)^2\right) = 0$$

We say Than $X_n \to X$ in square mean (in L^2).

X_n

$$\mathbb{P}(X_n = 0) = 1 - \frac{1}{n}$$

$$\mathbb{P}(X_n = 1) = \frac{1}{n}$$

$X = 0$ prob 1.

$$\mathbb{E}\left((X_n - X)^2\right) = 1^2 \cdot \frac{1}{n} \to 0$$

X_n

$$\mathbb{P}(X_n = 0) = 1 - \frac{1}{n}$$

$$\mathbb{P}(X_n = n^{1/3}) = \frac{1}{n}$$

$$\mathbb{E}\left((X_n - X)^2\right) = n^{2/3} \frac{1}{n} = n^{-1/3} \to 0$$

$$\mathbb{E}(X)^2 \leq \mathbb{E}(X^2)$$

if $X_n \to X$ in square mean

$$\mathbb{E}((X_n - X)^2) \to 0$$

$$\mathbb{E}(X_n - X)^2 \to 0$$

X_i are i.i.d r.v with

$\mathbb{E}(X_i) = \mu$ $\quad$ $var(X_i) = \sigma^2$

$$T_N = \frac{1}{N} \sum_{i=1}^{N} X_i$$

$\mathbb{E}(T_N) = \mu$ $\quad$ $var(T_N) = \frac{\sigma^2}{N}$

$$Var(T_N) = \mathbb{E}((T_N - \mu)^2) = \frac{\sigma^2}{N}$$

Th. if X_i are i.i.d r.v.

and $T_N = \frac{1}{N} \sum_i X_i$

$T_N \to \mu$ is square mean.

Square mean law of large numbers.

$X_n \quad X$

$$\lim_{n\to\infty} \mathbb{P}(|X_n - X| > \delta) = 0$$

$\forall \varepsilon\ \forall \delta\ \exists n$ such That

$$\mathbb{P}(|X_n - X| > \delta) < \varepsilon$$

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if $X_n \to X$ in square mean

Than $X_n \to X$ in prob.

Markov inequality if $X > 0$

$$\mathbb{P}(X > t) \le \frac{\mathbb{E}(X)}{t}$$

Chebyshev

Apply Markov To X^2

$$\mathbb{P}(X^2 > t) \leq \frac{\mathbb{E}(X^2)}{t}$$

Let X_n be a sequence of r.v.

assume $X_n \to X$ in square mean

$$\mathbb{P}(|X_n - X| > \delta) \leq \frac{\mathbb{E}((X_n - X)^2)}{\delta^2}$$

$$\lim \mathbb{P}(|X_n - X| > \delta) \leq$$

$$\frac{1}{\delta^2} \lim \mathbb{E}((X_n - X)^2) = 0$$

0

X_n $\qquad \mathbb{P}(X_n = 0) = 1 - \frac{1}{n}$

$$\mathbb{P}(X_n = n) = \frac{1}{n}$$

$X_n \to 0$ in probability

$$\mathbb{E}(X_n^2) = n$$

$$\mathbb{E}(X_n) = 1 \qquad \forall n$$

X_i is a measurement

X_i are i.i.d.

$$T_N = \frac{1}{N} \sum_{i=1}^{N} X_i \xrightarrow{P} \mathbb{E}(X_i) = \mu$$

$$\mathbb{P}(|T_N - \mu| > \delta) \leq \frac{\operatorname{var} X_i}{\delta^2 N}$$

if X_i $\quad \mathbb{P}(X_i = 1) = \frac{1}{2}$

$\mathbb{P}(X_i = -1) = \frac{1}{2}$

$$\sum_{i=1}^{100} X_i > 10 \implies$$

$$\mathbb{E}(X_i) = 0 \qquad \operatorname{Var}(X_i) = 1$$

$$\mathbb{P}\left(\left|\sum_{i=1}^{100} X_i\right| > 10\right) =$$

$$\mathbb{P}\left(|T_N| > 0.1\right) \leq \frac{1}{100 \cdot (0.1)^2} = 1$$

$N = 1000$

$$\mathbb{P}\left(|T_{1000}| > 0.1\right) \leq \frac{1}{1000 \cdot (0.1)^2} = \frac{1}{10}$$

X_i $\quad 2N \quad 2m$

$N+m \quad +1$

$N-m \quad -1$

$$S_{2N} = \sum_{i=1}^{2N} X_i$$

$$\mathbb{P}(S_{2N} = 2m) = \binom{2N}{2N-2m} 2^{-2N}$$

$$\frac{(2N)!}{(N-m)!\,(N+m)!}\;2^{-2N}$$

$$N! = \sqrt{2\pi}\, N^{N+\frac{1}{2}} e^{-N}$$

$$\frac{2^{-2N}\,(2N)^{2N}\,e^{-2N}}{(N-m)^{N-m}(N+m)^{N+m}\,e^{-(N-m)}\,e^{-(N+m)}}$$

$$\frac{N^{2N}}{N^{N-m}\left(1-\frac{m}{N}\right)^{N-m} N^{N+m}\left(1+\frac{m}{N}\right)^{N+m}}$$

$$= \frac{1}{\left(1-\frac{m}{N}\right)^{N-m}\left(1+\frac{m}{N}\right)^{N+m}}$$

$m = \delta N \qquad \delta \in [0,1]$

$$(1-\delta)^{(1-\delta)N}\,(1+\delta)^{(1+\delta)N}$$

$$(1-\delta^2)^{(1-\delta)N} (1+\delta)^{2\delta N} \to \infty$$

prob $m \simeq N$ is 0

$$m \simeq \sqrt{N}$$

$$\left(1+\frac{m}{N}\right)^{N+m} = e^{\log\left(1+\frac{m}{N}\right)(N+m)}$$